Children’s Mappings of Part-Whole Construct of Fractions

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The representation of part-whole relations that are embodied in fractional numbers continues to be a problematic area of learning for many children. In this study I examine this problem with a ten-year old child by analyzing his mappings between the language of fractions, area models and symbols. The visual models in this study were built and modified with computer manipulatives called *Javabars.* Results of analysis showed that the participant experienced difficulty in mapping symbolic representation of selected fractions to the area analogs.

Fractions provide teachers with insight into developments in children's understanding of numbers and relations among numbers. These understandings are built on both children's personal experiences, intuitions and formal knowledge taught in the classroom. Fractional numbers provide important prerequisite conceptual foundations for the growth and understanding of other number types and algebraic thinking in later years of their school and adult life. The complex nature of fractions continues to present difficulties for many young children in primary schools (Lamon, 1996; Mack, 2001; Anthony & Walshaw, (2003).

Significance of the Issue

The representation of part-whole relations that are embodied in fractional numbers continues to be a problematic area of learning for many children. The study of these representations continue to be an area of interest for researchers because children’s understanding of the part-whole relations directly impacts on their ability to unitise parts. While the emphasis on the development of multiple models for fractions has considerable support, there is a need to examine children’s reasoning with and the evolution of such models. Children who are able to build and connect different models of fractions can be considered to have developed a deep understanding of fractions as a class of numbers. Thus, whether children can relate parts of one model with that of the others is an important question for classroom practice and research. In a review of research in fraction understanding, Pitkethy and Hunting (1996) commented that

> Because the part-whole and ratio subconstructs have been shown to be fundamental to rational number development, further research in the area of initial fraction concepts is needed in order to show how the two subconstructs related in the growth of rational number understanding. (p. 34)

I examine the above problem by providing descriptions of mappings generated by a child working with the aid of a computer manipulative called *Javabars*. The principal research question is ‘What is the nature of mappings about part/whole relations that children could generate within *Javabars?’* The different mappings that are produced by the child are expected to inform researchers about how other children might connect whole number understandings with fraction number representations and the utility of computer-generated objects to assess the quality of children’s knowledge about fractions.

Conceptual Framework

An interdisciplinary approach is adopted in the present study in that work reported here draws on both cognitivist and socio-cultural perspectives about learning. The reasoning about fractions with concrete objects is analysed here in terms of the framework of structure mapping (Gentner, 1983). According to this framework, an analogy is a mapping from a base or source to a target. Elements in the base are mapped into elements in the target in such a way that relations in base and target correspond. Relations are mapped selectively, which means those relations that enter into a coherent structure. Levels of structure mapping can be distinguished by the complexity of the relations that are being mapped. From the socio-cultural perspective, the notion of zone of proximal development (ZPD), Vygotsky (1978), is invoked in the analysis as a way to explain how mapping as portrayed by Gentner can be elucidated between a child and a more senior member of the mathematics community.

Modelling of Part-whole Relations in Fractions

Modelling entails the development of constructs including part/whole constructs for fractions. During this process children need to map between fraction analog (area/ set models), the fraction name and their referents, and the fraction symbol. English and Halford (1995) analysed complexity of mathematical tasks in terms of dimensions. According to this analysis fractional numbers entail two dimensions, and analogical reasoning in this instance involves working with two dimensions. Thus the interpretation of fractions is seen to be more complex than that of whole numbers that are 1-dimensional entities. The modelling of fractional numbers can take many forms including mappings that children make between the dimensions and the objects that are used to show the mappings. The notion of 'x parts out of y equal parts' is one model of fractions. The relation between x and y needs to be mapped into the parts that are shaded in a figure. It is important that children make a distinction between shapes that are subdivided equally and those that are subdivided unequally before they can associate x and y in their reasoning about the links between these numbers and the corresponding parts and wholes in the figure. Children need to consider the numerator and denominator in relation to one another.

In the interpretation of relations shown in the area analog, children must consider a number of relations jointly in order to interpret the fraction represented by the analog. In essence, this involves a system mapping process. Before the children can determine the fraction represented by the shaded portion of the model, they must recognize that the parts are equal. They must then identify the total number of parts and map this number onto the name of fraction (e.g., eight equal parts – eighths). The number of parts shaded must then be identified. Determination of the fraction that is shaded involves coordinating both items of information to yield the fraction 3/8 (English & Halford, 1995, p. 130).

The area modelling of fractions constitutes a fraction analog. The concept of inclusion is entailed here because the shaded parts, together with the unshaded parts, are included in the whole. The salience of the whole in this analog means the unshaded parts will tend to be ignored.

Computer Generated Objects in the Development of Fraction Concepts

The complexities of fraction concepts have driven some researchers (Hunting, Davis & Pearn, 1996) to consider how best to design learning experiences involving computers that would assist children to demonstrate the p-w relations underlying fractions. Recently, a team of researchers from the University of Georgia has been investigating children's understanding of fractions with the aid of *Javabars* (Olive, 2000). The software has been primarily developed to examine the type of representations of fractions constructed by children. The software provides children with menus that help them draw bars of different shapes that can be modified in a number of ways. For instance, a given bar can be divided into equal or unequal parts that in turn could either be filled with different colours or isolated from the parent bar. Since its development, Javabars has been used to examine a range of learning issues that involve fractional numbers (Olive, 2002). In a more recent investigation, Olive (2003), analysed the on-screen actions of a third-grader in order to examine his strategies for simplifying and adding fractions. These strategies were argued to be based on the child's Generalised Number Sequence scheme which involved 'the transition from a 'ones' world to a world of composite units' (p.421), again drawing our attention to the notion of unitising.

Method

The depth-interviewing approach was used to collect data. Depth interviews are appropriate for field data-gathering processes designed to generate narratives that focus on specific research questions (Miller & Crabtree, 1999). This approach was used in the present study because it allowed the researcher to focus on data that were relevant to the questions of potential mappings that a child might construct with inputs while maintaining a degree of openness for the respondent.

Participants

A number of students from a suburban school in Australia volunteered to participate in the study. In order to highlight the persistence of learning problems, I report findings from interviews conducted with one of the participating children, Carl. Carl had studied whole numbers and fractions within the Number Strand of the New South Wales K-6 mathematics curriculum in the previous two years of primary school. At the time of the present study Carl had completed the topic on fractions in Year 5. Carl's teachers recommended that he was articulate and one who had difficulties with understanding relations between the whole numbers that appear in fractions.

Tasks and Procedure

The aim of study was to document knowledge about fractions and reasoning that children display within *Javabars*. Two fraction partition tasks were developed for the purposes of assessing children's knowledge and reasoning about fractions. The tasks focused on children's understanding of part-whole relationships, and how this understanding was represented in analogs generated by *Javabars*.

For the purposes of the first task (Problem 1), two bars were presented on the computer screen. The first bar on the left was not partitioned, representing a unit. The second bar was a copy of the first bar except that it had two features: a) it was divided into four equal segments, b) two of the segments were coloured in grey. Students were required to write the fraction for the grey part, and talk about it. The whole bar on the left-hand side was provided as an extra support for students to focus on the whole.

The second task (Problem 2) was similar to Problem 1. Again two bars were provided on the screen. The first bar on the left was the unit bar that could aid students' attempts to compare the wholes. The second bar (on the right) was identical to the first except that it was segmented into two unequal parts. The smaller part (blue) was one-seventh the size of the given unit bar. Students were asked to name two fractions that might be represented by the blue part. Further, each student was asked to test their conjectures about the fractional parts with the aid of *Javabars*. Students could activate the 'break' button on the menu and separate the smaller of the two segments. They could subsequently move this blue bar and align it along the unit bar or superimpose it on the unit bar. Alternatively, the students could carry out similar comparisons with the larger segment (red). The 'break', and 'copy' buttons on the computer screen could be used for the above moves, and students showed facility with these and related moves during the training session (see below) with an unrelated problem.

Carl was met individually for 90 minutes. During the first half of the interview the investigator introduced *Javabars* to the students and showed some of the basic features such as constructing a bar, colouring, breaking bars in equal and unequal parts (vertically/horizontally), and moving bars/pieces around the workspace within the computer screen. Comments from the participating students suggested that they found the activity enjoyable and easy to work with. The students were given time to experiment with *Javabars* by clicking the various buttons on the screen, and raise questions.

Reflections on the Mappings that can be Constructed

Both the problems provide rich contexts for children to demonstrate the construction of mappings. These mappings could reflect children's analyses of the parts and wholes embedded in the given bars at different levels. At one level, one could expect children to identify the parts as chunks without consideration to subparts or the equal size of the subparts. That is, in Problem 1 children could 'see' two parts (grey and red) and both these as being of same size. These constitute legitimate mappings between the various parts of the bar with notions about colour and space.

There are also a number of other mappings that underpin the elucidation of part-whole relations and their symbolic equivalents. Students have to reason that the given bar is a whole, and that this whole has been divided into four equal parts. That is, students have to recognize that the bar on the right is the same as the one on the left of the screen. This relationship can be established by visual inspection but *Javabars* provides a more exact way to assess this. That is, students could use the *Copy* button to make a copy of the left-hand bar and then align this with the right-hand bar. This action should lead them to reason that they are working with the same whole. Secondly, students have to recognize that the right-hand bar is divided equally into four parts. Again, while a visual inspection could provide an intuitive answer, we expected that students would use the Parts menus to break the bar vertically into 4 equal parts using the appropriate menus in the software. This should result in the deductions that there are four fourths and the two of the fourths are colored in grey. The answer could be 2/4 or half (1/2).

While Problem 2 again involves the elucidation of parts from a whole, the reasoning involved more chains of mappings than in the previous problem. A visual inspection would reveal that the blue part could be five, six or seven times that size of the remainder of the right-hand bar (red) leading to the conclusion that blue is $1/5^{th}$ and red is $4/5^{th}$ of the whole and so on. Even in this approach students have to map the symbols to the parts and wholes of the given whole bar on the left on the screen. A more systematic reasoning for this problem could involve students breaking the blue part and aligning this under the red part in order to ascertain how many of the blue parts would make the red part. Students have to replicate the blue parts by using the Copy button on the screen. Six of the blue parts should be sufficient to make the red bar. From this point students should be able to conclude that the blue part constitutes one-sixth of the red, and then finally that the blue is in fact $1/7^{th}$ of the whole bar. Alternatively, they could reason that the red is $6/7^{th}$ of the whole bar.

Results

Table 1 shows the dialogue between the researcher (R) and Carl as he attempted Problem 1. At C2 (Table 1), Carl was able to correctly count the number of grey parts in Bar 1B. He could not relate this to the four parts into which the whole bar is divided.

Table 1
Carl (C) - Context 1, R - Researcher

Speaker and Utterance Number	Utterance
C1	Write the fraction for bar 1B that is shaded in grey (difficulty in pronouncing ‘fraction’).
R1	This is the bar 1B. Can you give an answer?
C2	Two
R2	Anything else?
C3	No

Dialogue in Table 2 shows that Carl was able to construct a number of mappings. At C2, he was able to recognize the two coloured parts and their relative size. That is, there is evidence of two types of mappings. In the first instance, Carl is able to map the blue and red parts with the blue and colours respectively. He was also reason spatially and map the smaller with the blue part and red part with the larger part of the bar. While he hesitated, with support from the researcher, the above levels of mappings were extended to examining parts within parts. C13 and C15 demonstrate an attempt to draw out the part-whole relations between the blue and red parts. C16 shows Carl was validating his intuitive understanding of the relative size of the red and blue parts.

Discussion and Implications

The nature of the problems presented in this study were such that in order to respond correctly Carl needed to map four representations of fractions: part/whole relations, language, symbols and bars (unit and partitioned). A deeper understanding of fractions was required in order to ‘see’ the links among these representations, and the embodiment of the part-whole relation within each representation. That is, the integration of the different representations constitutes a key characteristic of the quality of knowledge that supports Carl’s understanding and interpretation of fractions. This feature of the knowledge base that emphasises connectedness or organization has been argued to facilitate better integration and use of prior knowledge of mathematics (Chinnappan, 1998; Prawat, 1989; Schoenfeld, 1992). A more structured knowledge base for fractions is also necessary for further developments in identifying the multiplicative relation between the numerator and denominator. For example, a recent study conducted by Mack (2001) about operations involving fractions led her to suggest that failure to transfer symbolic understanding of fractions to the concept of partitioning could impede students' ability to perform multiplication and divisions operations involving fractions. Here one could see the conceptual value of establishing links between partitioning of fractional numbers and their symbols.

Table 2
Carl (C) - Context 2, R - Researcher

Speaker and Utterance Number	Utterance
C1	Name two fractions that might be represented by the bar 1c.
R1	Can you see bar 1c? What can you see there? (R showing bar 1c on the computer screen)
C2	One blue skinny one and half red
R2	You are asked to write the fraction. You have the blue skinny one as you say and the red one, right?
C3	Yes
R3	Question is can you express blue as a fraction of the red?
C4	Um…
R4	Which is bigger here?
C5	The red
R5	Can you express the fractions? Do you understand the word fraction?
C6	To say what's bigger and do what's skinny
R6	Ok, how many of the blue will make up the red?
C7	Five
R7	Any other possibility?
C8	Um..
R8	Remember, we can break this and move this around. Would you like to try that yourself?
C9	I do not know what to do with the red
R9	Do you wish to break the figure like break it and move it around?
C10	Break it up
R10	Now you have broken this, the blue and the red. Do you wish to take them apart?
C11	Ya
R11	What do you want to do next?
C12	Break the red up
R12	Why
C13	So we know how many pieces of them there are
R13	Ok. How do you want to break the red?
C14	That way (Indicating breaking top to bottom)
R14	(R helps Carl to break up)
C15	(Clicking number 5 on the 'Parts' button)
R15	So you have the red one into 5 parts. Why did you break this into 5 parts and not say, 10 parts?
C16	Because to see if I was right.

Overall, the child in the present study had developed some understanding of fractions. Carl could exploit the dynamic bar features provided by *Javabars* in a number of conceptually powerful ways. Firstly, Carl they could align the partitioned bar with the unit bar in order to make judgments about the relative size of the partitions and produce the correct numerical forms. He showed a degree of comfort in breaking and assembling parts to make the whole (unitizing), and interpreting the result in terms of fractions. This was particularly in evidence during his solution of Problem 2. Carl could shift from one representation (one-sixth) to another without too much effort. This apparently seamless transfer among representations is indicative of the robustness of his fraction schemas. In their analysis of mathematical understanding, English and Halford (1995) argued that the mapping of elements of one representation with elements in a different representation induces cognitive load, and that one way to reduce this load would be to improve the strength of the links among knowledge components in the schema. Thus it would seem that the robustness of Carl's schema helped him decrease the cognitive load associated with the mapping process. The reduction in cognitive load would account for the ease with which he could move across representations.

In his study of fractions strategies, Olive (2003) reported that the two participating children were able to make a whole bar given a part(s) of the bar (e.g. 2/7th). We see evidence of similar actions in the present study as Carl manages to use the blue parts to construct the whole bar. While he was not explicit about the ratio between the parts, breaking the red portion into five parts suggests an interesting line of reasoning. While the pattern of actions reported here seem to be consistent with those of Olive (2003), the mapping analyses provide a different angle for our interpretations of part-whole understandings.

Behr, Harel, Post and Lesh (1992) argued that recording children's understanding could be used to make instructional decisions. While this is a reasonable suggestion, more qualitative data are needed on this issue. While it is too early to generalise on the basis of the actions of one student, the results do seem to suggest that teachers need to become familiar with level of children's knowledge of fractions before a software such as *Javabars* could be introduced either as a learning or teaching tool.

Also intuitive thinking plays a key role in mathematical thinking and learning. I argue that this constitutes a natural way for children to reason about part-whole constructs in their understanding of fractions. While this might appear to be sufficient, as teachers we need to emphasize other ways for children to model and test the validity of these representations. It is suggested that this evaluation involves reasoning and that the use of analogs for reasoning provides an effective way to examine the nature of interpretations constructed by children.

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